Sparse thresholding for regularization, interpolation, and dealiasing

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SUMMARY

Sparse thresholding algorithms are useful for interpolating data acquired with random spatial sampling patterns because the sampling creates incoherent aliasing artefacts that are lower in amplitude than the underlying signal. Regular sampling patterns, on the other hand, create coherent aliasing artefacts with amplitudes that are difficult to distinguish from signal. Historically, interpolation algorithms have specifically targeted either random or regular sampling patterns. Unfortunately, seismic data are rarely acquired using purely regular or purely random layouts, so a flexible solution is desirable. This abstract introduces Sparse Thresholding for Regularization, Interpolation, and Dealiasing (STRIDE). STRIDE is able to suppress both random and regular aliasing artefacts by iteratively extracting the dominant dip from the gradient of the objective function.

INTRODUCTION

Seismic data are rarely acquired with sufficient spatial sampling to meet the requirements of prestack migration, making prestack interpolation a critical processing step. Traditionally, seismic data have been acquired using the most regular patterns possible for sources and receivers. Numerous methods have been proposed to improve the sampling of these data. For example, upsampling by a factor of two can be achieved using prediction error filters computed at half frequencies (Spitz, 1991). More recently, attention has shifted toward randomized acquisition methods such as compressive sensing that promise better sampling at a reduced cost (Donoho, 2006). Unlike regularly sampled data, randomly sampled data are interpolated using methods that suppress incoherent aliasing artefacts in a transform domain. Practically it is difficult to acquire data using a perfectly regular or perfectly random grid of sources and receivers. Sampling patterns often vary regionally within a dataset, or are different for each dimension. For example, seismic sources are often more restricted in their placement than receivers, leading to an irregular pattern of sources recorded by a regular pattern of receivers. In such cases a more flexible interpolation method is required.

Sparse approximations to seismic data were introduced long before compressive sensing formalized the concept of random sampling (see for example Thorson and Claerbout (1985)). Sparse thresholding algorithms or "greedy" approaches such as the Anti-Leakage Fourier Transform (Xu et al., 2005), Projection Onto Convex Sets (Abma and Kabir, 2006), or Matching Pursuit (Øzbek et al., 2009) were proposed to interpolate random patterns of missing traces. These approaches are effective at generating highly sparse approximations to the data, but they often take many iterations to converge to a solution and they are unable to interpolate regular patterns of missing traces. Minimum Weighted Norm Interpolation (MWNI) (Liu and Sacchi, 2004) converges more quickly than sparse thresholding based approaches by iteratively bootstrapping the power spectral density (PSD) of the complete data via Iteratively Re-weighted Least Squares (IRLS). While this approach accurately fits the data using relatively few iterations, it generally does not provide solutions that are as sparse as those provided by greedy solvers.

This abstract introduces a new method, Sparse Thresholding for Regularization, Interpolation, and Dealiasing (STRIDE), that is able to suppress regular and random aliasing artefacts by incorporating amplitude and dip information into a sparse thresholding operator. STRIDE iteratively constructs an estimate of the PSD of the data, and uses this estimate to efficiently solve for a sparse model that fits the data. The model can then be used to predict data at missing trace locations (interpolation), to populate a regular grid (regularization), or to upsample to a finer grid (dealiasing).

METHOD

Sparse Thresholding for Regularization, Interpolation, and Dealiasing (STRIDE) seeks to minimize the objective function

$$J = ||\mathbf{d} - \mathbf{TFm}||_2^2 + \lambda||\mathbf{W}^{-2}\mathbf{m}||_2^2 \quad (1)$$

where $\mathbf{d}$ are the observed data in the FX domain, $\mathbf{m}$ is the unknown model in the FK domain, $\mathbf{F}$ is the forward multidimensional Fourier transform and $\mathbf{T}$ could either be implemented as a sampling operator that weights live trace locations by 1 and missing trace locations by 0, or as a local interpolator that maps from indexed grid points to irregular spatial positions (Carozzi and Sacchi, 2021). Parameter λ controls the trade-off between data fitting and the sparsity of the solution, while matrix $\mathbf{W}$ promotes sparsity to supress aliasing in the solution. Like MWNI, the weighting requires some prior knowledge about the PSD of the complete data.

To interpolate random spatial sampling patterns the PSD of individual frequency slices can be reliably estimated from the data via IRLS. Interpolating regular sampling patterns, on the other hand, requires information from multiple frequencies to be incorporated into the solution. Numerous methods have been proposed to extract this information from the data. Curry (2010) applied low pass filtering along radial lines in the FK domain to create a weighting function that supresses aliased energy, while a number of subsequent methods

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proposed constructing a dip spectrum to identify the dominant dips in the data (Naghizadeh, 2012; Gao et al., 2013; Curry et al., 2017; Qin et al., 2018). A dip spectrum provides a relative coherence measure of different dips in the data. In STRIDE, a dip spectrum is computed at every iteration of the solution using the gradient of the objective function, $\mathbf{g}$. The gradient is projected onto the dominant dip, p, and combined with the current model to create an estimate of the PSD of the complete data, which is then used to update the model. A pseudocode for the proposed method is given below:

function STRIDE($\mathbf{d}$,$N_{iteration}$,α,$\boldsymbol{\beta}$,λ) {

$\mathbf{m} := \mathbf{0}$

for $j = 1 : N_{iteration}$ {

$$\mathbf{g} := \mathbf{F}^{*}\mathbf{T}^{*}\Big(\mathbf{d} - \mathbf{TFm}\Big)$$

$$p := dominant\_dip(\mathbf{g})$$

$$\hat{\mathbf{g}} := threshold(\mathbf{g},p,\beta_j)$$

$$\mathbf{W} := |\mathbf{m} + \alpha\hat{\mathbf{g}}|$$

$$\mathbf{m} := \min_{\mathbf{m}}\Big(||\mathbf{d} - \mathbf{TFm}||_2^2 + \lambda||\mathbf{W}^{-2}\mathbf{m}||_2^2\Big)$$

}

return $\mathbf{m}$

}

Here α specifies a step size, and $\boldsymbol{\beta}$ is an iteration dependant amplitude threshold. The function *dominant_dip()* sums along radial fans in the FK domain to identify the dominant dip, p, while the function *threshold()* masks a single radial fan in FK domain corresponding to this dip and applies amplitude thresholding to further promote sparsity in the solution. In every iteration of STRIDE the model, $\mathbf{m}$, is updated by minimizing equation 1 via Conjugate Gradients. Using this approach, the estimate of the PSD improves in accuracy as the residual becomes small.

EXAMPLES

The first example illustrates the application of STRIDE on 2D synthetic data. The FK display in Figure 1 shows incoherent aliasing artefacts as a result of randomly decimating 50% of the data. In this case either dip or amplitude information could be used to identify signal. Figure 2 shows the result of applying 3 iterations of STRIDE to interpolate the data, using both dip and amplitude to constrain the solution. The interpolation has achieved a high level of accuracy.

Next, interpolation is applied to regularly decimated data. The FK display in Figure 3 shows coherent FK domain aliasing artefacts as a result of decimating every second trace. In this case it is only possible to identify signal using dip information. Figure 4 shows the result of applying 3 iterations of STRIDE to interpolate the data, using both dip and amplitude to constrain the solution. The interpolation has again achieved a high level of accuracy with the exception of two small artifacts visible in the FK display at a normalized frequency of approximately 0.1. These artefacts correspond to points where coherent aliasing artefacts overlapped with signal in the decimated data. At these points neither dip nor amplitude information were sufficient to distinguish signal from aliasing. This example illustrates the benefit of acquisition methods that reduce the coherence or intensity of aliasing artefacts (Hennenfent and Herrmann, 2008; Naghizadeh, 2015).

Interestingly, a variety of operations generate similar artefacts in the FK domain. Figure 5 shows artefacts generated by regularly (left column) or randomly (right column) varying the wavelet (top row), statics (middle row), or interference (bottom row). This suggests that STRIDE could be used to solve a variety of problems beyond interpolation including deconvolution, statics, or deblending data acquired with a mixture of regular or random shooting times.

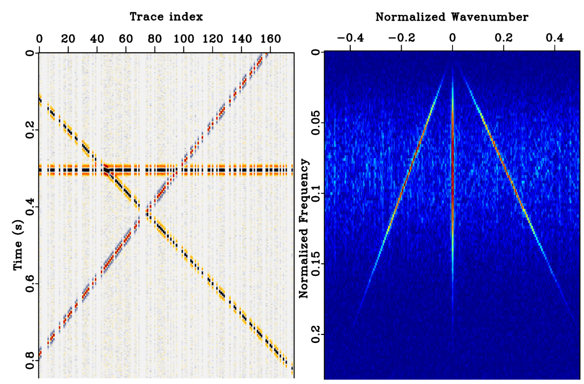

Figure 1: Randomly decimated data in the TX (left) and FK (right) domains. Note the incoherence of the aliasing artefacts in the FK domain.

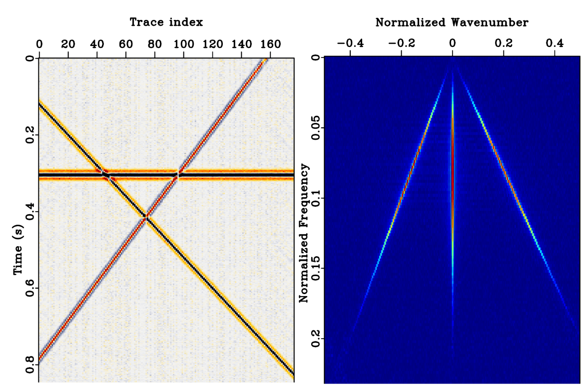

Figure 2: STRIDE interpolation of randomly decimated data in the TX (left) and FK (right) domains.

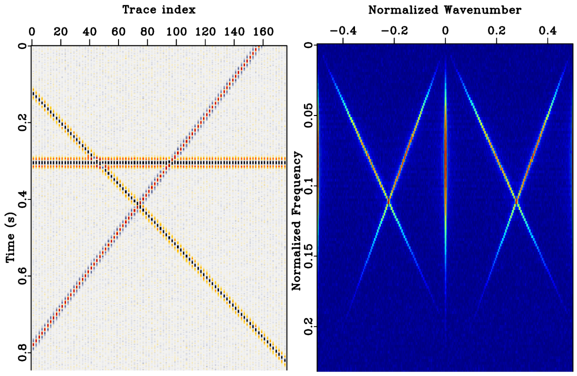

Figure 3: Regularly decimated data in the TX (left) and FK (right) domains. Note the coherence of the aliasing artefacts in the FK domain.

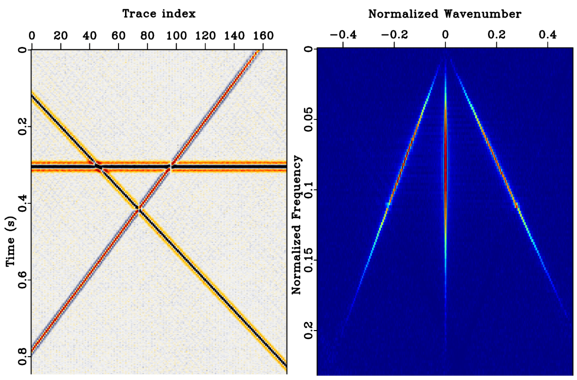

Figure 4: STRIDE interpolation of regularly decimated data in the TX (left) and FK (right) domains.

Finally, STRIDE is used to interpolate the Teapot Dome 3D Dataset. Figure 6 shows a cross-spread gather sorted by source/receiver before interpolation. Within this cross-spread the source spacing is erratic while the receivers are more regularly spaced (approximately every second bin is empty). STRIDE is able to accurately interpolate the data despite the fact that the dimensions are sampled so differently.

CONCLUSIONS

Seismic data are rarely acquired using purely regular or purely random sampling patterns, so a flexible interpolation method is often desireable. STRIDE is able to supress both random and regular aliasing artefacts with high accuracy by incorporating information from multiple frequencies into a sparse thresholding based solver.

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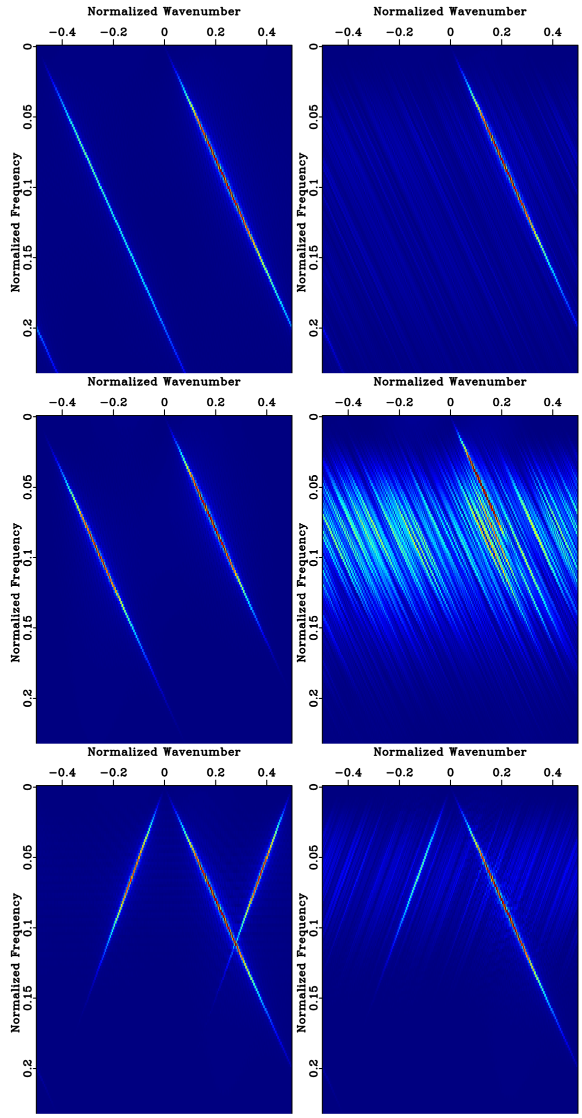

Figure 5: A single dip in the FK domain with artefacts generated by regularly (left column) or randomly (right column) varying the wavelet (top row), statics (middle row), or interference (bottom row)

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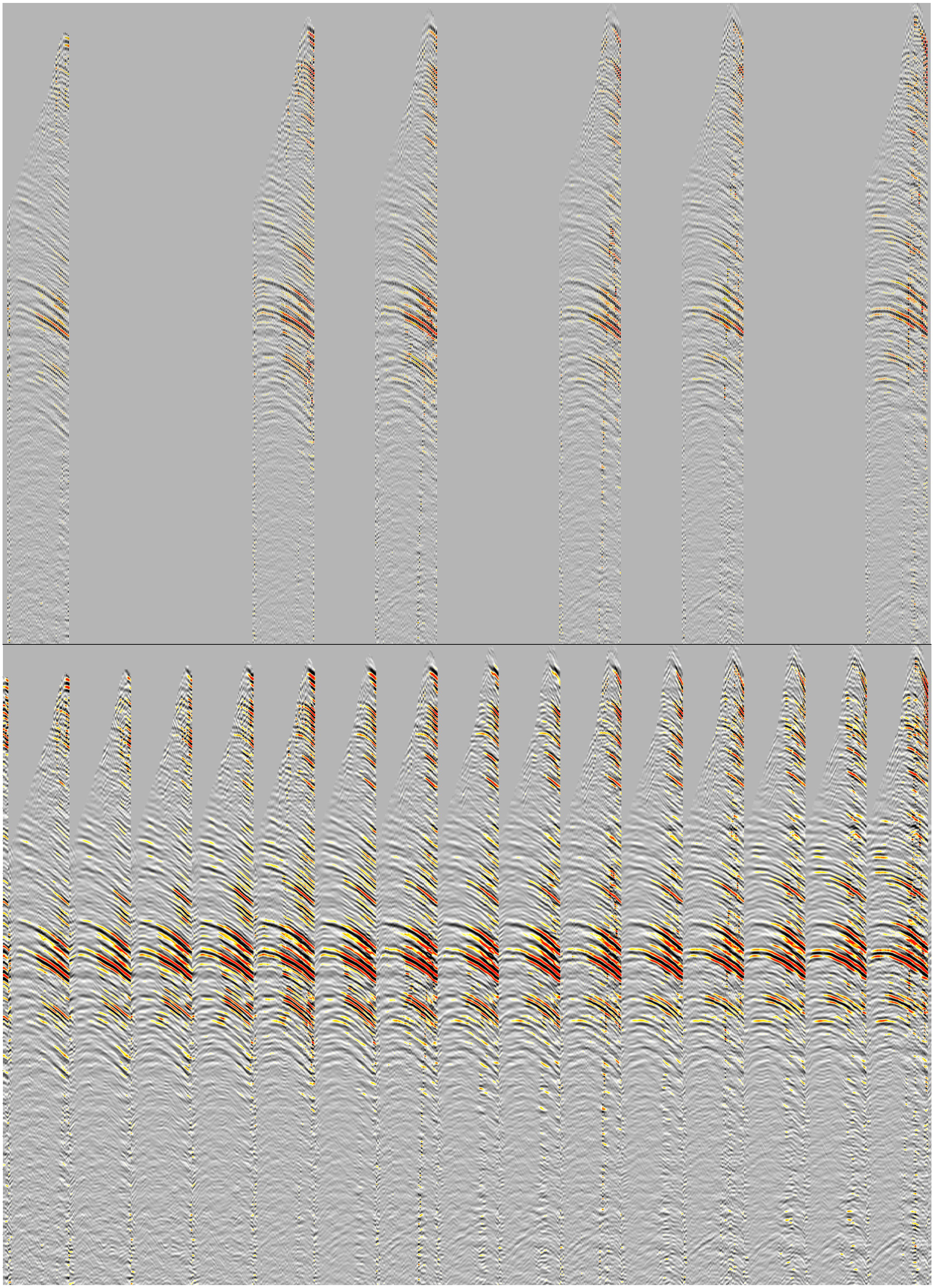

Figure 6: A cross-spread gather from the Teapot Dome 3D Dataset sorted by source/receiver before and after interpolation. Within this cross-spread the receiver spacing is regular while the source spacing is more erratic.

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